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Comment on: "First-order phase transitions of type-I superconducting films" [Phys. Lett. A 322 (2004) 111]

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The theory of fluctuation induced first order phase transition from normal to uniform (Meissner) superconducting state in a zero external magnetic field was developed for thin films of type I superconductors in a series of papers [1, 2, 3, 4, 5, 6, 7]. The same problem was considered in a recent theoretical investigation [8]. The authors of the paper [8] derived an erroneous effective free energy of superconducting slabs (films) and this error led them to entirely wrong conclusions. Here we shall make clear the genesis of this mistake by using the notations in [8].

At a certain stage of derivation of the effective free energy $\mathcal{F}(\phi)$ as a function of the modulus ($\phi \equiv |\psi|$) of the superconducting order parameter ψ, the authors of Ref. [8] have obtained the following result

$$\frac{d\mathcal{F}(\phi)}{d\phi} = m_0^2\phi + \frac{\lambda}{2}\phi^3 + (d-1)e^2\phi I(\phi), \tag{1}$$

where $m_0^2 = (T/T_0 - 1)$ and λ are well known Landau parameters of the ϕ^4-theory of second order phase transitions [9]. The spatial dimensionality is denoted by d, $e = |e|$ is the electron charge, and

$$I(\phi) = (2\pi)^{-\frac{d}{2}}\left[2^{-\frac{d}{2}}\Gamma\left(1-\frac{d}{2}\right)(e\phi)^{d-2} + 2\sum_{n=1}^{\infty}\left(\frac{e\phi}{nL}\right)^{\frac{d}{2}-1}K_{\frac{d}{2}-1}\left(e\phi Ln\right)\right] \tag{2}$$

(cf Eqs. (5) and (11) in [8]). In (2), $K_\nu(z)$ is the MacDonald function [10]. All quantities in (1) and (2) are dimensionless and, in particular lengths, such as the film thickness L, are scaled by the zero-temperature coherence length ξ_0 ($L \to L/\xi_0$) [8].

The compact notation

$$R_n^\nu(\phi) = (e\phi Ln)^\nu K_\nu\left(e\phi Ln\right), \tag{3}$$

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will be used in the remainder of this Comment. The integral formula

$$\int_0^y x^{\frac{d}{2}} K_{\frac{d}{2}-1}(x)dx = -y^{\frac{d}{2}} K_{\frac{d}{2}}(y) + 2^{\frac{d}{2}-1}\Gamma\left(\frac{d}{2}\right), \tag{4}$$

is valid for dimensionalities $d > 0$ [10], and we shall apply the known expansion of the function $K_\nu(z)$ for small values of z (see, e.g., [11]).

In order to obtain the derivative $d\mathcal{F}/d\phi$ to the relevant order ϕ^3, one has to expand the second term in r.h.s. of Eq. (2) to order ϕ^2. The results should be valid for spatial dimensionalities $2 < d < 4$. The authors of [8] have performed an expansion of the second term in r.h.s. of Eq. (2) to order ϕ^0 and failed to reveal a term of order ϕ^{d-2} that exactly cancels the first term in r.h.s. of (2). The expansion contains also a term of type ϕ^2 which gives a fluctuation contribution to the ϕ^4-invariant in the free energy as shown below. This means that the free energy $\mathcal{F}(\phi)$ does not contain a ϕ^d-term for $2 < d < 4$ and, hence, this free energy does not describe a first order phase transition at all.

Here we show the mentioned error in [8] by the calculation of the free energy $\mathcal{F}(\phi)$ to order ϕ^4. From (1) and (2) we obtain

$$\mathcal{F} = \frac{m_0^2}{2}\phi^2 + \frac{\lambda}{8}\phi^4 + (d-1)e^2 J(\phi), \tag{5}$$

where the integral

$$J(\phi) = \int_0^\phi d\varphi \varphi I(\varphi), \tag{6}$$

is calculated using (4). With the help of (3) $J(\phi)$ can be written in the form

$$J(\phi) = (2\pi)^{-\frac{d}{2}}\left[\frac{2^{-\frac{d}{2}}}{d}\Gamma\left(1-\frac{d}{2}\right)e^{d-2}\phi^d + \frac{2^{\frac{d}{2}}\Gamma(d/2)\zeta(d)}{e^2L^d} - \frac{2}{e^2L^d}\sum_{n=1}^{\infty}\frac{R_n^{d/2}(\phi)}{n^d}\right]. \tag{7}$$

Now the expansion of $R_n^{d/2}(\phi)$ for $(e\phi Ln) \ll 1$ straightforwardly leads to the following form of the sum in (7):

$$\sum_{n=1}^{\infty}\frac{R_n^{d/2}}{n^d} = 2^{\frac{d}{2}-1}\Gamma(d/2) \left[\zeta(d) + \frac{\zeta(d-2)y^2}{2(2-d)} + \frac{\zeta(d-4)y^4}{8(4-d)(2-d)} + \frac{\Gamma(-d/2)\zeta(0)y^d}{2^d\Gamma(d/2)}\right], \tag{8}$$

where $y = (e\phi L)$, and $\zeta(z)$ is the (Riemann) zeta function [11]. Inserting the result (8) in (7) and having in mind that $\zeta(0) = -1/2$ one easily obtains that the first two terms in r.h.s. of (7) are cancelled by two new terms of type ϕ^0 and ϕ^d which drop from the expansion of the third term in r.h.s. of (7). We shall not show this elementary calculation.

Therefore, the fluctuation correction $\Delta\mathcal{F} = (d-1)e^2J(\phi)$ to $\mathcal{F}(\phi)$ does not contain any ϕ^d term; in particular, ϕ^3 term for $d = 3$. Thus, the proof that the effective free energy, given by Eq. (14) in [8] does not follow from the Eqs. (5) and (11) in the same paper [8] is completed.

Our present analysis shows that the fluctuation contributions to the free energy in [8] should be given by

$$\Delta\mathcal{F}(\phi) = \frac{2^{\frac{d}{2}-1}(d-1)\Gamma(d/2)}{\pi^{d/2}(d-2)L^d}\left[\zeta(d-2)(e\phi L)^2 + \frac{\zeta(d-4)}{4(4-d)}(e\phi L)^4\right], \tag{9}$$

For $d \to 3$ the first term in (9) is divergent. To avoid this singularity the authors of [8] performed a renormalization. In this way they achieved a finite negative shift $(-e^2 T_0/L)$ of the bulk characteristic temperature T_0. The shift vanishes in the bulk limit $L \to \infty$, as should be, but the second term in (9), which has been overlooked in [8], tends to large negative values for $L \gg 1$ and $d < 4$. When $L \to \infty$, the magnitude of the large negative value of the fluctuation contribution exceeds the value of the positive parameter λ and the superconducting phase becomes unstable. In such a situation there is no phase transition, or, alternatively, terms of order in ϕ higher than fourth should be included in the free energy with the aim to ensure the description of tricritical and/or other multicritical points and another type of first order phase transitions [9]. In the present case, this scenario is impossible because all terms of order ϕ^a $(a > 4)$ generated by (7) tend to infinity for $L \to \infty$. This simply means that the method used in [8] is unreliable.

The error in the derivation of Eq. (14) in [8] totally invalidates the thermodynamic analysis performed in the second part of [8] and intended to describe the thermodynamic properties of (almost) three dimensional slabs. Besides, several new incorrect points of view are introduced in this thermodynamic analysis. For example, the correct equilibrium phase transition temperature has not been calculated and, moreover, the latter has been wrongly identified with the temperature which nullifies the coefficient in front of the ϕ^2-term. Let us emphasize that the temperature, at which the coefficient of the ϕ^2-term becomes equal to zero, is never the phase transition temperature of a first-order phase transition produced by a ϕ^3-term (see, e.g., Ref. [9]). In their thermodynamic analysis the authors of [8] missed to take advantage of the thorough consideration of three-dimensional (bulk) superconductors presented in Ref. [7].

Finally we wish to stress that the authors [8] have made a quite inappropriate comparison of their results with the results by our coauthors and ourselves [1, 4, 5, 6]. Our results are intended to describe quasi-two-dimensional films, where the magnetic fluctuations affect the phase transition properties by an entirely different mechanism, namely, by a free energy term of the type $|\psi|^2\ln|\psi|$. This low-dimensional limit cannot be achieved by a simple analytical continuation of results obtained for dimensionalities $2 < d < 4$.

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